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Improved treatment of Fresnel equations

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Abstract. Present treatments of the Fresnel equations are criticised. A full treatment without the defects is presented, the amplitudes and phases of the electric fields being calculated, and curves displayed for a variety of cases.

1. Introduction

The Fresnel equations, which describe the behaviour of an electromagnetic wave at a plane surface, have a fundamental importance in any study of electromagnetism, and also have many practical applications.

They are straightforward in principle, though not without subtleties in their detailed analysis. Nevertheless it is extremely surprising that there appears to be no treatment available which covers all cases in an adequate manner. Many works (e.g. Jeans 1925, Jackson 1962, Lorrain and Corson 1970), while presenting general analysis of the dielectric–dielectric boundary, specialise, for the dielectric–metal boundary, to the assumptions of normal incidence or high conductivity.

The treatments usually referred to as standard–those of Stratton (1941) (to be referred to as S) and Born and Wolf (1965)— consider the metal–metal boundary for general angle of incidence and conductivity, but these treatments, which are virtually identical, are unsatisfactory as a number of quantities are incompletely defined, and the presence of these causes the expressions for the amplitudes and phases of the reflected and refracted waves to be ambiguous. It might also be mentioned that the form in which these authors present their results is extremely complex, so that their use is far from straightforward.

There seems therefore a need for a treatment such as that given in this paper, in which expressions are obtained for the various fields which are completely unambiguous. Although lengthy they are absolutely straightforward and are completely general, applying for example to the case of total internal reflection.

The technique has been used by Whitaker (1978) in the specific application of the Fresnel equations to the theory of angular photoemission. In the present paper the general analysis is presented, which includes magnetic media, and concentrates on reflected and refracted rays rather than Cartesian components of the total vector potential.

2. Conventions used in electromagnetism

It may be worth briefly reviewing the conventions used in electromagnetism, as a correct use of the convention chosen is vital in what follows.

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The initial choice is the trivial one between time factors of the form of $\exp(-i\omega t)$ and $\exp(+i\omega t)$. The form of a plane wave moving in the $+z$ direction is then $\exp(\pm ikz \mp i\omega t)$ and, since k is equal to $(\omega\bar{n}/c)$, where $\bar{n}$ is the complex refractive index, to ensure attenuation $\bar{n}$ must be written as $n_1 \pm in_2$ with n_2 positive, and $\bar{\epsilon}$ as $\epsilon_1 \pm i\epsilon_2$ with ϵ_2 positive. Here we take time factors of the form $\exp(-i\omega t)$, and so the dielectric constant is written as $\epsilon_1 + i\epsilon_2$.

3. The complex angle of refraction

Our notation is as follows. The radiation is incident on the surface of a metal of dielectric constant $\epsilon_{1m} + i\epsilon_{2m}$, and permeability μ_m, from a dielectric of dielectric constant ϵ_d and permeability μ_d. The angle of incidence is θ_d. The z axis is along the normal to the surface, z increasing as we move into the dielectric. The plane of incidence is the y–z plane, and the incident wave vector has a component along $+y$.

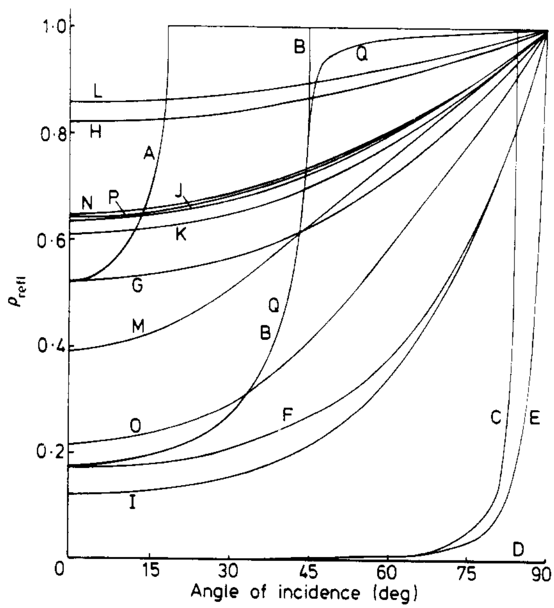

Figure 1. Values of $\rho_{\rm refl}$ for s polarisation as a function of angle of incidence. The values of ϵ_m in this and the other graphs are: A 0·1; B 0·5; C 0·99; D 1; E 1·01; F 2; G10; H 100; I 1+0·5i; J 1+10i; K 10+10i; L 100+100i; M 0·1+0·5i; N 0·1+10i; O 0·5+0·5i; P 0·5+10i; Q 0·5+0·01i.

The magnitudes of the wave vectors in dielectric and metal are given by

$$k_d = (\omega/c)\epsilon_d^{1/2}\mu_d^{1/2}$$
$$k_m^2 = (\omega^2/c^2)(\epsilon_{1m} + i\epsilon_{2m})\mu_m. \tag{3.1}$$

The refracted wave must have the form

$$\exp(-ik_m z \cos\theta_m + ik_m y \sin\theta_m).$$

The product $k_m \sin\theta_m$ is easy to evaluate, as by Snell's Law it is equal to $k_d \sin\theta_d$ and is

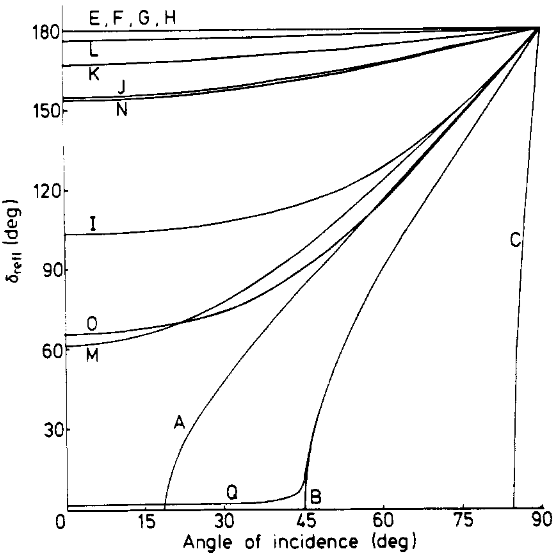

Figure 2. Values of δ_{refl} for s polarisation. Curve P is omitted as it lies between J and N.

thus real. Using Snell's Law again, we find that $k_{\text{m}} \cos \theta_{\text{m}}$ is equal to

$$(\omega/c)(\epsilon_{1\text{m}}\mu_{\text{m}} + \text{i}\epsilon_{2\text{m}}\mu_{\text{m}} - \epsilon_{\text{d}}\mu_{\text{d}} \sin^2 \theta_{\text{d}})^{1/2}$$

but further consideration must be given to the choice of root of this complex number.

At this point Stratton (1941, p 501) writes $\cos \theta_{\text{m}}$ as $\rho \exp(\text{i}\gamma)$ and gives expressions for ρ and γ. However, γ is defined only as modulo π by his equations, and so he does not distinguish between two values of $\cos \theta_{\text{m}}$ differing by a factor of -1. Thus his quantities p and q, real linear combinations of $\rho \cos \gamma$ and $\rho \sin \gamma$, are defined only in magnitude and not in sign (Stratton 1941, p 503). The values of the components of the electromagnetic fields subsequently found by Stratton are thus not determined uniquely.

We put

$$k_{\text{m}} \cos \theta_{\text{m}} = (\omega/c)S^{1/4}[\cos(\phi/2) + \text{i} \sin(\phi/2)] \tag{3.2}$$

where

$$S = (\epsilon_{1\text{m}}\mu_{\text{m}} - \epsilon_{\text{d}}\mu_{\text{d}} \sin^2 \theta_{\text{d}})^2 + \epsilon_{2\text{m}}^2\mu_{\text{m}}^2 \tag{3.3}$$

$$\cos \phi = (\epsilon_{1\text{m}}\mu_{\text{m}} - \epsilon_{\text{d}}\mu_{\text{d}} \sin^2 \theta_{\text{d}})/S^{1/2}$$
$$\sin \phi = \epsilon_{2\text{m}}\mu_{\text{m}}/S^{1/2}. \tag{3.4}$$

Since $\epsilon_{2\text{m}}$ is positive, we know that $\pi \geqslant \phi \geqslant 0$. There are therefore two possible ranges for $\phi/2$: $\pi/2 \geqslant \phi/2 \geqslant 0$, and $3\pi/2 \geqslant \phi/2 \geqslant \pi$.

There are two conditions on $k_{\text{m}} \cos \theta_{\text{m}}$. First, so that the wave has a component in the $+z$ rather than the $-z$ direction, the real part must be positive. Secondly, for the wave to be attenuated, the imaginary part must be positive. Both considerations lead to the range for $\phi/2$ being $\pi/2 \geqslant \phi/2 \geqslant 0$.

In equation (3.2), therefore, we need

$$\cos(\phi/2) = [S^{1/2} + (\epsilon_{1m}\mu_m - \epsilon_d\mu_d \sin^2\theta_d)]^{1/2}/(S^{1/4}2^{1/2})$$
$$\sin(\phi/2) = [S^{1/2} - (\epsilon_{1m}\mu_m - \epsilon_d\mu_d \sin^2\theta_d)]^{1/2}/(S^{1/4}2^{1/2}). \quad (3.5)$$

This value of $k_m \cos\theta_m$ will now be used in the Fresnel equations. (Only the product is needed; k_m and $\cos\theta_m$ themselves can each only be defined to within a factor of -1.)

4. General form of Fresnel equations

For each wave we initially define time-independent quantities; for example the electric field of the incident wave is written as

$$\boldsymbol{E}_0 \exp(-\mathrm{i}k_d z \cos\theta_d + \mathrm{i}k_d y \sin\theta_d)\exp(-\mathrm{i}\omega t).$$

$\boldsymbol{E}_0$ is independent of time; for each mode of polarisation, one direction must be defined as the positive direction. For s polarisation ($\boldsymbol{E}$ normal to the plane of incidence) E_0, and the analagous quantities E_{refr} and E_{refl}, are defined as positive when $\boldsymbol{E}_0$, $\boldsymbol{E}_{refr}$ and $\boldsymbol{E}_{refl}$ are in the $+x$ direction. B_0 and B_{refr} are positive when the corresponding vectors have components in the $-y$ and $-z$ directions; for B_{refl}, the components must be in the $+y$ and $-z$ directions.

For p polarisation ($\boldsymbol{E}$ in the plane of incidence), it is convenient to define B_0, B_{refr} and B_{refl} as positive when the vectors lie in the $+x$ direction. E_0 and E_{refr} are positive when the vectors have components in the $+y$ and $+z$ directions; for E_{refl}, the components must be in the $-y$ and $+z$ directions.

(Though these considerations may seem tedious and elementary, without explicit definitions algebraical results are meaningless; the diagrams of Stratton (1941, pp 493–494), for instance, seem less than completely satisfactory.)

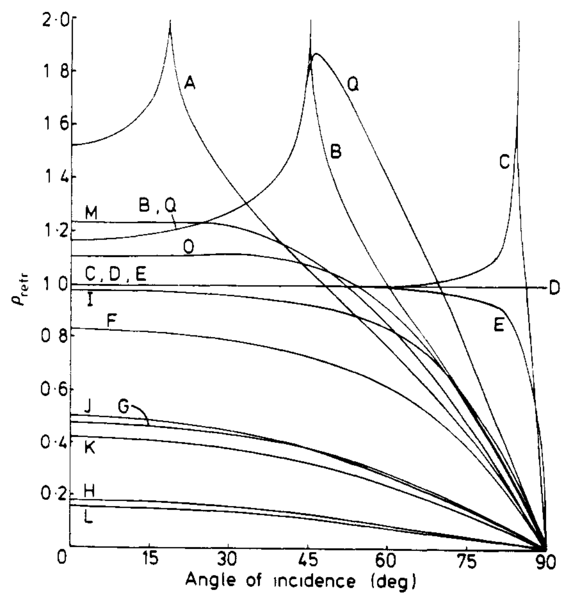

Figure 3. Values of ρ_{refr} for s polarisation. Curves N and P are omitted as they lie extremely close to J.

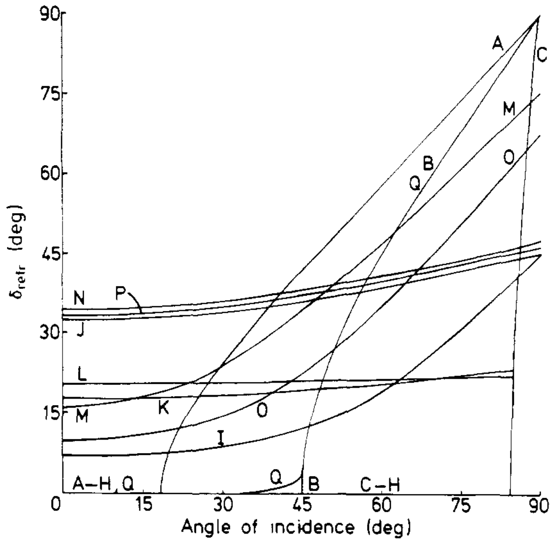

Figure 4. Values of δ_{refr} for s polarisation.

4.1. s polarisation

Applying the boundary conditions in the conventional manner, we find for E_{refl} in this case

$$E_{refl}/E_0 = (\mu_m^2\epsilon_d \cos^2\theta_d - \mu_d S^{1/2} - i2^{1/2}\mu_m\mu_d^{1/2}\epsilon_d^{1/2}u_- \cos\theta_d)/t_1 \tag{4.1}$$

where

$$t_1 = \mu_m^2\epsilon_d \cos^2\theta_d + \mu_d S^{1/2} + 2^{1/2}\mu_m\mu_d^{1/2}\epsilon_d^{1/2}u_+ \cos\theta_d \tag{4.2}$$

$$u_\pm = [S^{1/2} \pm (\epsilon_{1m}\mu_m - \epsilon_d\mu_d \sin^2\theta_d)]^{1/2}. \tag{4.3}$$

If E_{refl} is written as

$$E_{refl} = E_0\rho_{refl}\exp(-i\delta_{refl}) \tag{4.4}$$

then

$$\rho_{refl} = (\mu_m^2\epsilon_d \cos^2\theta_d + \mu_d S^{1/2} - 2^{1/2}\mu_m\mu_d^{1/2}\epsilon_d^{1/2}u_+ \cos\theta_d)^{1/2}/t_1^{1/2} \tag{4.5}$$

$$\tan\delta_{refl} = \frac{2^{1/2}\mu_m\mu_d^{1/2}\epsilon_d^{1/2}u_- \cos\theta_d}{\mu_m^2\epsilon_d\cos^2\theta_d - \mu_d S^{1/2}} \tag{4.6}$$

$$\cos\delta_{refl} = \frac{\mu_m^2\epsilon_d\cos^2\theta_d - \mu_d S^{1/2}}{[\mu_m^4\epsilon_d^2\cos^4\theta_d + \mu_d^2 S - 2\mu_d\mu_m^2\epsilon_d\cos^2\theta_d(\epsilon_{1m}\mu_m - \epsilon_d\mu_d\sin^2\theta_d)]^{1/2}}. \tag{4.7}$$

The corresponding quantity for the refracted wave, E_{refr}, is given by

$$E_{refr}/E_0 = 2\mu_m\epsilon_d^{1/2}\cos\theta_d[(\mu_m\epsilon_d^{1/2}\cos\theta_d + 2^{-1/2}\mu_d^{1/2}u_+) - i2^{-1/2}\mu_d^{1/2}u_-]/t_1. \tag{4.8}$$

With similar definitions as in equation (4.4)

$$\rho_{refr} = 2\mu_m\epsilon_d^{1/2}\cos\theta_d/t_1^{1/2} \tag{4.9}$$

$$\tan\delta_{refr} = \mu_d^{1/2}u_-/(2^{1/2}\mu_m\epsilon_d^{1/2}\cos\theta_d + \mu_d^{1/2}u_+) \tag{4.10}$$

$$\cos\delta_{refr} = (\mu_m\epsilon_d^{1/2}\cos\theta_d + 2^{-1/2}\mu_d^{1/2}u_+)/t_1^{1/2}. \tag{4.11}$$

4.2. p polarisation

For this case, $E_{\rm refl}$ is given by

$$E_{\rm refl}/E_0 = \{[\mu_d\mu_m(\epsilon_{1m}^2+\epsilon_{2m}^2)\cos^2\theta_d - \mu_m\epsilon_d S^{1/2}] + i[2^{1/2}\mu_d^{1/2}\epsilon_d^{1/2}\cos\theta_d u_-(S^{1/2}-\epsilon_d\mu_d\sin^2\theta_d)]\}/t_3 \quad (4.12)$$

where

$$t_3 = \mu_d\mu_m(\epsilon_{1m}^2+\epsilon_{2m}^2)\cos^2\theta_d + \mu_m\epsilon_d S^{1/2} + 2^{1/2}\mu_d^{1/2}\epsilon_d^{1/2}u_+\cos\theta_d(S^{1/2}+\epsilon_d\mu_d\sin^2\theta_d). \quad (4.13)$$

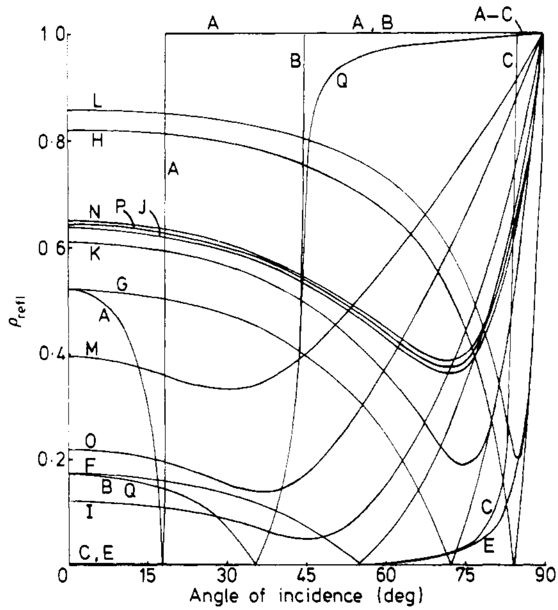

Figure 5. Values of $\rho_{\rm refl}$ for p polarisation.

With similar definitions again

$$\rho_{\rm refl} = \{\mu_d\mu_m(\epsilon_{1m}^2+\epsilon_{2m}^2)\cos^2\theta_d + \mu_m\epsilon_d S^{1/2} - 2^{1/2}\mu_d^{1/2}\epsilon_d^{1/2}u_+\cos\theta_d(S^{1/2}+\epsilon_d\mu_d\sin^2\theta_d)\}^{1/2}/t_3^{1/2}. \quad (4.14)$$

$$\tan\delta_{\rm refl} = \frac{-2^{1/2}\mu_d^{1/2}\epsilon_d^{1/2}u_-\cos\theta_d(S^{1/2}-\epsilon_d\mu_d\sin^2\theta_d)}{\mu_d\mu_m(\epsilon_{1m}^2+\epsilon_{2m}^2)\cos^2\theta_d - \mu_m\epsilon_d S^{1/2}} \quad (4.15)$$

$$\cos\delta_{\rm refl} = [\mu_d(\epsilon_{1m}^2+\epsilon_{2m}^2)\cos^2\theta_d - \epsilon_d S^{1/2}]/\{\mu_d^2\cos^4\theta_d(\epsilon_{1m}^2+\epsilon_{2m}^2)^2 + \epsilon_d^2 S + 2\mu_d\epsilon_d\cos^2\theta_d[-\epsilon_{1m}\mu_m(\epsilon_{1m}^2+\epsilon_{2m}^2) + \epsilon_d\mu_d\sin^2\theta_d(\epsilon_{1m}^2-\epsilon_{2m}^2)]\}^{1/2}. \quad (4.16)$$

Discussion of the electric field of the refracted wave is a little more complex in this case. (Of course, if magnetic rather than electric fields are calculated, the modes of polarisation are interchanged apart from signs and constants.) It is convenient to start

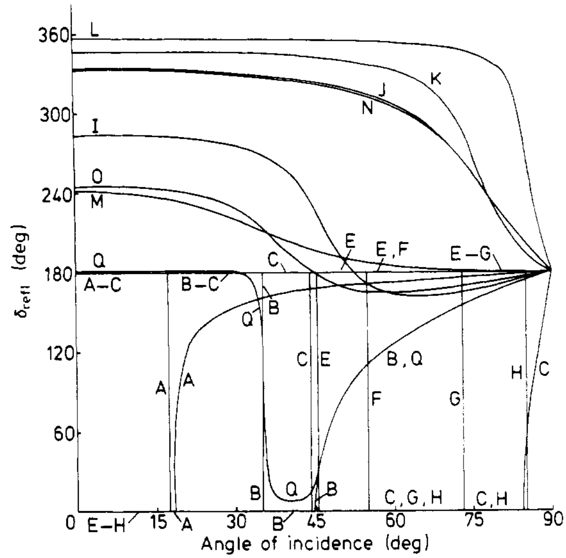

Figure 6. Values of δ_{refl} for p polarisation. Curve P is omitted as it lies between J and N.

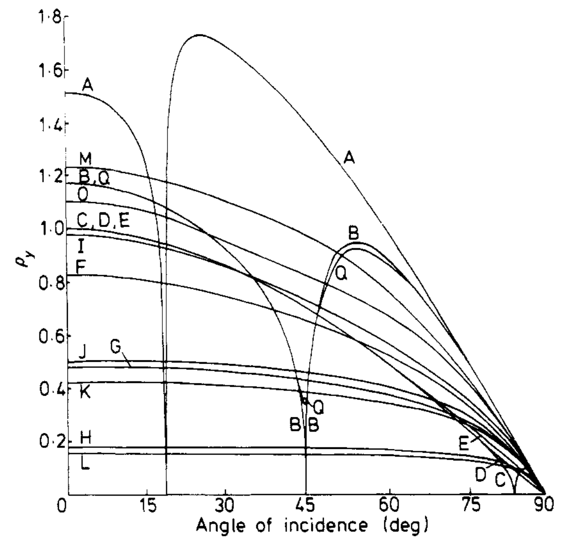

Figure 7. Values of ρ_y for p polarisation. Curves N and P are omitted as they lie extremely close to J.

from the refracted magnetic induction given by

$$\frac{B_{x(\text{refr})}}{B_0}=\frac{2\mu_m(\epsilon_{1m}+i\epsilon_{2m})\cos\theta_d}{\mu_d(\epsilon_{1m}+i\epsilon_{2m})\cos\theta_d+\mu_d^{1/2}\epsilon_d^{1/2}S^{1/4}[\cos(\phi/2)+i\sin(\phi/2)]}$$
$$\times\exp[-iS^{1/4}\cos(\phi/2)(\omega/c)z]\exp[S^{1/4}\sin(\phi/2)(\omega/c)z]$$
$$\times\exp[ik_dy\sin\theta_d]\exp(-i\omega t). \tag{4.17}$$

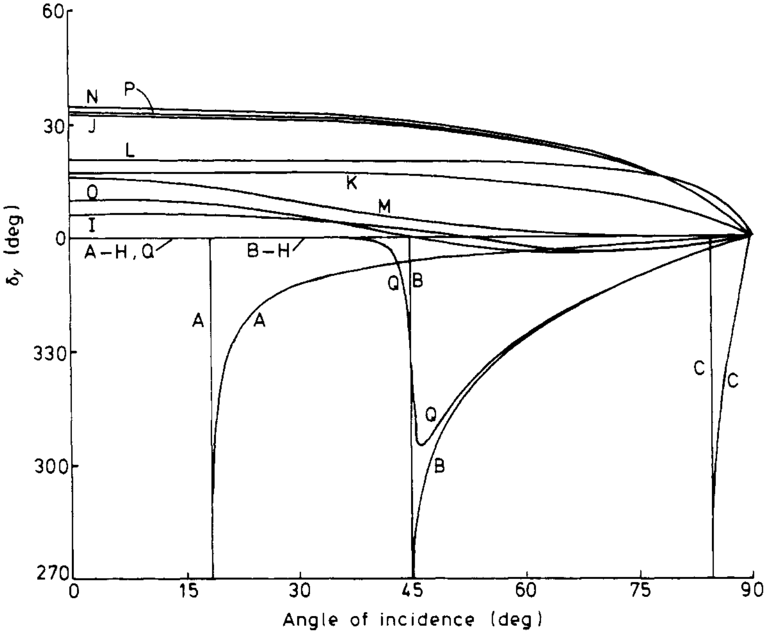

Figure 8. Values of δ_y for p polarisation.

From Maxwell's equations the components of the electric field in the metal may be obtained. Because these are not in phase, they cannot be combined in a satisfactory way into an expression for a refracted wave, and will be given individually:

$$E_y/E_0 = \{2\mu_m\epsilon_d S^{1/2} + 2^{1/2}\mu_d^{1/2}\epsilon_d^{1/2}\cos\theta_d[u_+(S^{1/2}+\epsilon_d\mu_d\sin^2\theta_d) + iu_-(\epsilon_d\mu_d\sin^2\theta_d - S^{1/2})]\}\cos\theta_d/t_3 \tag{4.18}$$

$$\rho_y = \{4\epsilon_d\mu_m\cos^2\theta_d S^{1/2}/t_3\}^{1/2} \tag{4.19}$$

$$\tan\delta_y = \frac{\mu_d^{1/2}\cos\theta_d u_-(S^{1/2}-\epsilon_d\mu_d\sin^2\theta_d)}{2^{1/2}\mu_m\epsilon_d^{1/2}S^{1/2}+\mu_d^{1/2}u_+\cos\theta_d(S^{1/2}+\epsilon_d\mu_d\sin^2\theta_d)} \tag{4.20}$$

$$\cos\delta_y = [\mu_d^{1/2}u_+(S^{1/2}+\epsilon_d\mu_d\sin^2\theta_d)\cos\theta_d + 2^{1/2}\epsilon_d^{1/2}S^{1/2}\mu_m]/\{2^{1/2}S^{1/4}\mu_m^{1/2}[\mu_d\mu_m\cos^2\theta_d(\epsilon_{1m}^2+\epsilon_{2m}^2) + 2^{1/2}\mu_d^{1/2}\epsilon_d^{1/2}u_+\cos\theta_d(S^{1/2}+\epsilon_d\mu_d\sin^2\theta_d)+\epsilon_d\mu_m S^{1/2}]^{1/2}\} \tag{4.21}$$

$$E_z/E_0 = 2\mu_d^{1/2}\mu_m\epsilon_d\cos\theta_d\sin\theta_d[(\mu_d^{1/2}\epsilon_{1m}\cos\theta_d+\epsilon_d^{1/2}2^{-1/2}u_+) - i(\mu_d^{1/2}\epsilon_{2m}\cos\theta_d+\epsilon_d^{1/2}2^{-1/2}u_-)]/t_3. \tag{4.22}$$

$$\rho_z = 2\epsilon_d\mu_d^{1/2}\mu_m^{1/2}\cos\theta_d\sin\theta_d/t_3^{1/2} \tag{4.23}$$

$$\tan\delta_z = (\epsilon_d^{1/2}u_- + 2^{1/2}\mu_d^{1/2}\epsilon_{2m}\cos\theta_d)/(\epsilon_d^{1/2}u_+ + 2^{1/2}\mu_d^{1/2}\epsilon_{1m}\cos\theta_d) \tag{4.24}$$

$$\cos\delta_z = \mu_m^{1/2}(\mu_d^{1/2}\epsilon_{1m}\cos\theta_d + 2^{-1/2}u_+\epsilon_d^{1/2})/t_3^{1/2} \tag{4.25}$$

5. Special cases

These formulae are completely general, and reduce in special cases to the simpler formulae.

5.1. $\epsilon_{2m}=0$

For the case where ϵ_{2m} is zero (ϵ_{1m} being positive), there are two types of situation. If $\epsilon_{1m}\mu_m > \epsilon_d\mu_d \sin^2\theta_d$ then

$$S^{1/2}=\epsilon_{1m}\mu_m-\epsilon_d\mu_d\sin^2\theta_d \tag{5.1}$$

$$\cos(\phi/2)=1, \qquad \sin(\phi/2)=0 \tag{5.2}$$

$$u_+=2^{1/2}S^{1/4}, \qquad u_-=0. \tag{5.3}$$

then, for *s* polarisation

$$\rho_{refr}=2\mu_m\epsilon_d^{1/2}\cos\theta_d/[\mu_m\epsilon_d^{1/2}\cos\theta_d+\mu_d^{1/2}(\epsilon_{1m}\mu_m-\epsilon_d\mu_d\sin^2\theta_d)^{1/2}] \tag{5.4}$$

$$\delta_{refr}=0 \tag{5.5}$$

$$\rho_{refl}=|\mu_m\epsilon_d^{1/2}\cos\theta_d-\mu_d^{1/2}S^{1/4}|/(\mu_m\epsilon_d^{1/2}\cos\theta_d+\mu_d^{1/2}S^{1/4}). \tag{5.6}$$

δ_{refl} is equal to zero if $\mu_m^2\epsilon_d\cos^2\theta_d+\mu_d^2\epsilon_d\sin^2\theta_d>\epsilon_{1m}\mu_m\mu_d$, but there is a phase change of π if this inequality is not obeyed.

For *p* polarisation

$$\rho_{refl}=|\mu_d^{1/2}\epsilon_{1m}\cos\theta_d-\epsilon_d^{1/2}S^{1/4}|/(\mu_d^{1/2}\epsilon_{1m}\cos\theta_d-\epsilon_d^{1/2}S^{1/4}). \tag{5.7}$$

δ_{refl} is zero if $\mu_d\epsilon_{1m}^2\cos^2\theta_d+\epsilon_d^2\mu_d\sin^2\theta_d>\epsilon_d\epsilon_{1m}\mu_m$ and there is a phase change of π otherwise.

For the refracted wave, E_y and E_z are given by

$$E_y/E_0=[2\epsilon_d^{1/2}\cos\theta_d(\epsilon_{1m}\mu_m-\epsilon_d\mu_d\sin^2\theta_d)^{1/2}]/t_4 \tag{5.8}$$

$$E_z/E_0=2\epsilon_d\mu_d^{1/2}\cos\theta_d\sin\theta_d/t_4 \tag{5.9}$$

where

$$t_4=\mu_d^{1/2}\epsilon_{1m}\cos\theta_d+\epsilon_d^{1/2}(\epsilon_{1m}\mu_m-\epsilon_d\mu_d\sin^2\theta_d)^{1/2}. \tag{5.10}$$

It is easy to show that E_y and E_z combine in this case to give a refracted wave at the appropriate angle given by Snell's Law.

If, however, $\epsilon_{1m}\mu_m<\epsilon_d\mu_d\sin^2\theta_d$, we are in the region of total internal reflection. Here

$$S^{1/2}=\epsilon_d\mu_d\sin^2\theta_d-\epsilon_{1m}\mu_m \tag{5.11}$$

$$\cos(\phi/2)=0, \qquad \sin(\phi/2)=1 \tag{5.12}$$

$$u_+=0, \qquad u_-=2^{1/2}S^{1/4} \tag{5.13}$$

From equation (3.2), and the form of the refracted wave in § 3, we see that the amplitudes of the field will indeed decay exponentially as we move into the 'metal'. It is obvious that $\rho_{refl}=1$, and ρ_{refr}, δ_{refl} and δ_{refr} may be found easily.

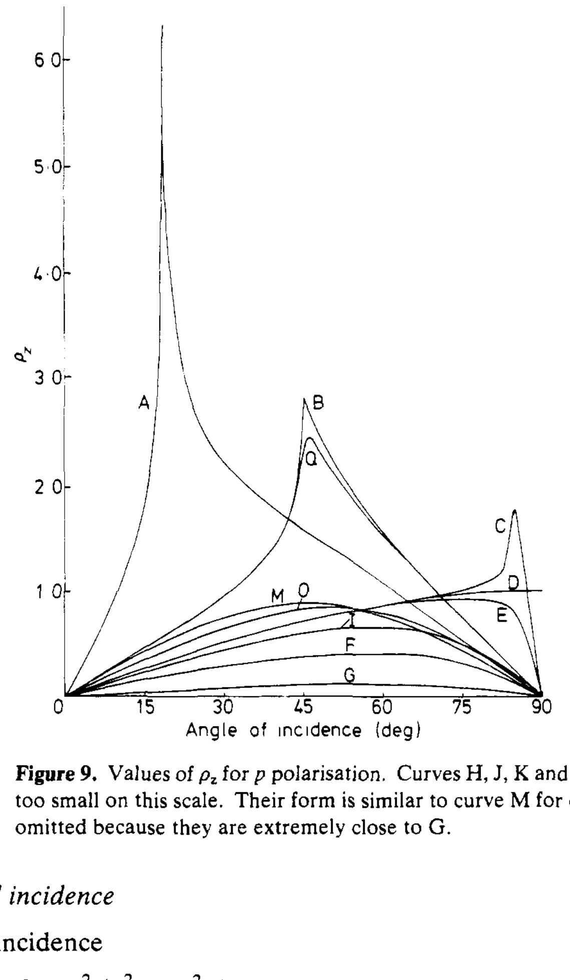

Figure 9. Values of ρ_z for p polarisation. Curves H, J, K and L are omitted as all values are too small on this scale. Their form is similar to curve M for example. Curves N and P are omitted because they are extremely close to G.

5.2. Normal incidence

For normal incidence

$$S = \mu_m^2(\epsilon_{1m}^2 + \epsilon_{2m}^2) \tag{5.14}$$

$$u_\pm = \mu_m^{1/2}[(\epsilon_{1m}^2 + \epsilon_{2m}^2)^{1/2} \pm \epsilon_{1m}]^{1/2} \tag{5.15}$$

The interesting point about this case is that the two modes of polarisation should become identical. Calculation of the various quantities shows that this is indeed the case, except for δ_{refl} where there is a difference of π between the two expressions. This is a direct result of our choices of positive direction at the beginning of § 4. For p polarisation, the positive direction for E_{refl} is opposite to that for E_0 in this case, whereas for s polarisation the directions are the same. There is no similar difficulty for the refracted wave if ρ_y and δ_y are defined as ρ_{refr} and δ_{refr}.

5.3. High conductivity

In the case of high conductivity we have

$$S = \epsilon_{2m}^2\mu_m^2 \tag{5.16}$$

$$\cos(\phi/2) = \sin(\phi/2) = 2^{-1/2} \tag{5.17}$$

$$u_+ = u_- = \epsilon_{2m}^{1/2}\mu_m^{1/2}. \tag{5.18}$$

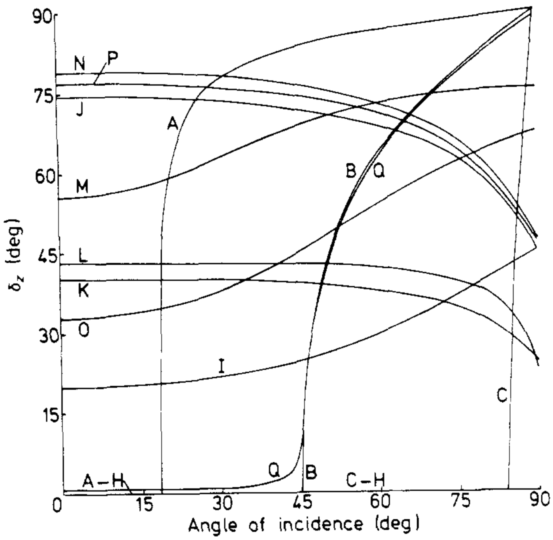

Figure 10. Values of δ_z for p polarisation.

All the usual formulae may be obtained. For the case of s polarisation, ρ_{refr} is very small, while ρ_{refl} is only just less than unity:

$$\rho_{\text{refr}} = (2\mu_{\text{m}}^{1/2}\epsilon_{\text{d}}^{1/2} \cos\theta_{\text{d}})/(\mu_{\text{d}}^{1/2}\epsilon_{2\text{m}}^{1/2}) \tag{5.19}$$

$$\rho_{\text{refl}} = 1 - [2^{3/2}\mu_{\text{m}}^{1/2}\epsilon_{\text{d}}^{1/2} \cos\theta_{\text{d}}/\mu_{\text{d}}^{1/2}\epsilon_{2\text{m}}^{1/2}] \tag{5.20}$$

δ_{refr} is $\pi/4$, while δ_{refl} is π. Similar results are obtained for p polarisation.

6. Conclusion

The various quantities have been evaluated for a number of cases. (ϵ_{d}, μ_{d} and μ_{m} are put equal to unity.) A great variety of behaviour is displayed, although it could still not be claimed that the coverage is complete. No treatment in any way as comprehensive as this appears to be available.

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